Numerical Solution of Partial Differential Equations– Finite Difference Method

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The Finite Difference Method is one of very important and powerful numerical methods for solving partial differential equations in science and engineering. In this course, we develop the theoretical foundation and algorithm. Roughly speaking, it transforms a PDE problem to the problem of solving a system of coupled algebraic equations. The domain of the independent variables is approximated by a discrete set of points called a grid, and the dependent variables are defined only at these points. First, we will introduce the method for one-dimensional problems so that we can learn the essential tools that carry over to higher dimensions. Topics include elliptic equations and elliptic system (Elasticity, Maxwell′s equations), parabolic (heat equations), wave equations and Navier Stokes system and hyperbolic equations (Conservation laws and Hamilton Jacobi equation). Also, we introduce the so-called immersed interface method for the interface problems and multi-physics PDE models. Essential tools are difference approximations of derivatives and integration the function space approximation theory (Lax-Milgram formulation). We study the stability analysis and the convergence and error analysis. The students will be able to implement finite difference methods using sample Matlab codes.

- Basic PDE models and Difference approximations. Numerical ODEs.
- Central Difference approximations and Higher order methods.
- Iterative Methods for Sparse Linear Systems
- Theoretical Foundation of Finite Difference Methods for elliptic, parabolic, hyperbolic equations.
- Operator splitting methods for time integration.
- Immersed interface .methods for discontinuous media and Domain decomposition.
- Specific Topics (Request is considered).

A good learning environment is essential to the success of a class and Grade is based on:

- Homework (analytic part and computer projects) about every two weeks: 30%
- Two take-home tests: 60% (time to be announced)
- Class participation and Quiz 10% (Essential for Study and good grade)

Reference Text:

(1) Finite Difference Methods for Ordinary and Partial Differential Equations Steady State and Time Dependent Problems, SIAM, Randall J. LeVeque.

(2) Li, Z.L. and Ito, K. (2006) The Immersed Interface Method. Numerical Solutions of PDEs Involving Interfaces and Irregular Domains. SIAM, Philadelphia.

Office Hours: TTh 1:00-2:00 p.m., otherwise Appointment.